Physics II
ISI B.Math
Mid Semestral Exam: September 11, 2019

Total Marks: 80
Time: 3 hours
Answer all questions

1. **(Marks : 8 $\times$ 2 = 16)**

State whether the following statements are TRUE or FALSE in general with a very brief (a few lines) explanation for your choice.

i) An adiabatic transformation is also isentropic.

ii) A reversible engine with a high efficiency is particularly well suited to work as a refrigerator when run in reverse.

iii) For one mole of gas $T\left(\frac{\partial S}{\partial T}\right)_P - T\left(\frac{\partial S}{\partial T}\right)_V = R$

iv) The entropy of a system always increases when it undergoes an irreversible transformation.

v) An ideal gas Carnot engine operates between the temperatures T_h and T_l. If the ideal gas in the same engine was replaced by a photon gas with equation of state $P = \frac{1}{3}\sigma T^4$ where σ is a positive constant, the efficiency would be lower because now the pressure is independent of volume.

vi) An ideal gas violates the third law of thermodynamics.

vii) A liquid in equilibrium with its saturated vapour is enclosed in an cylinder attached to a piston. The piston is slowly raised to a final position. As a result 5 gms of the liquid gets converted to 5gms of vapour while the temperature and pressure remain constant. If G is the Gibbs potential, F is the Helmholtz potential and S the entropy of the liquid vapour system, in this process G and F remain the same while S increases.

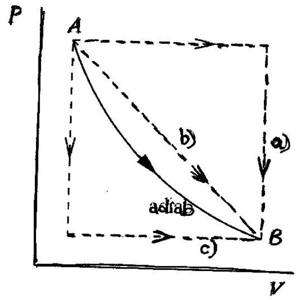
viii) An exact differential expression relating thermodynamic variables is given by

$$dB = CdE - FdG + HdJ$$

$P = B - FG - CE$ can qualify as a new thermodynamic potential function consistent with the expression for dB.

2. **(Marks: 4 + 12 = 16)**

(i) Show that $\left(\frac{\partial U}{\partial P}\right)_V = \frac{C_V\kappa}{\beta}$ where the symbols have their usual meanings.



(ii) During a quasistatic reversible transformation from $A \to B$ during which no heat is exchanged with the environment, the pressure of a certain amount of gas is found to change with the volume according to the equation $P = \alpha V^{-\frac{5}{3}}$ where α is a constant. Find the work done on the system and the net amount of heat absorbed by the gas in each of the following three transformations which take the system from state A to state B as indicated in the diagram by a) , b) and c). You may assume that the transformations are quasi-static and reversible. Express your results in terms of P_A, P_B, V_A and V_B (the constant α should not appear in your results.)

3. **(Marks: 6 + 4 + 6 = 16)**

(i) One mole of nitrogen gas at pressure P_i and temperature T_i in a rigid cylinder of volume V is raised to $T_f (T_f > T_i)$ by placing it in contact with an infinite reservoir at temperature T_f. You can assume that nitrogen is an ideal gas. What are the changes in entropy of the gas and the universe ? Show that the change is irreversible.

(ii) Assuming the same initial conditions and access to the reservoir at temperature T_f, now allow one end of the cylinder to act like a piston. Describe a means of raising the gas temperature to T_f (with the final volume of V) without increasing the entropy of the universe.

(iii) Now attach an identical evacuated cylinder (of same volume V) to the rigid cylinder containing nitrogen (at temperature T_i and pressure P_i) such that they share a common diathermic wall which acts as a partition between the two chambers. Arrange to thermally insulate the double cylinder system from the surroundings. What is the change of entropy of the universe when the common wall is removed ?

4. **(Marks: 4 + 6 + 6 = 16)**

(i) Show that the entropy S increases with volume V for a gas whose pressure p is known to be proportional to its absolute temperature T when the volume is held constant.

(ii) Prove the following inequalities

$$\left(\frac{\partial S}{\partial P}\right)_H < 0; \left(\frac{\partial S}{\partial V}\right)_U > 0$$

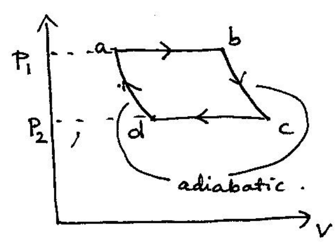
where U, H, S, P and V are the internal energy, enthalpy ,entropy, pressure and volume respectively. Use the inequalities to establish that the following processes are irreversible (a) a gas makes a free adiabatic expansion from volume V to volume $V + dV$. (b) A gas expands adiabatically by the Joule-Thompson throttling process from p to $p + dp$ where $dp < 0$.

(iii) The equation of state of a new form of matter is $p = AT^3/V$ where A is a constant. The internal energy of the system is given by

$$U = BT^n \ln\left(\frac{V}{V_0}\right) + f(T)$$

where B, V_0, n are constants. $f(T)$ is a function of T. Find B and n. [Hint: Use the fact that entropy S is a state function]

5. **(Marks: 6 + 5 + 5 = 16)**



Consider an engine working in a reversible cycle and using an ideal gas with a constant heat capacity C_P as a working substance. The cycle consists of two processes at constant pressure, joined by two adiabatics.

(a) Find the efficiency of the engine in terms of p_1 and p_2 and $\gamma = \frac{C_P}{C_V}$.

(b) Which temperature of T_a, T_b, T_c, T_d is the highest, and which is the lowest ?

(c) Show that a Carnot engine working with the same gas between the highest and lowest temperatures has greater efficiency than this engine.

Information you may (or may not) need

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$